

Vortex reconnections between coreless vortices in binary condensates

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Abstract. Vortex reconnections plays an important role in the turbulent flows associated with the superfluids. To understand the dynamics, we examine the reconnections of vortex rings in the superfluids of dilute atomic gases confined in trapping potentials using Gross-Pitaevskii equation. Further more we study the reconnection dynamics of coreless vortex rings, where one of the species can act as a tracer.

Keywords: Vortex, superfluids, reconnections

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INTRODUCTION

In superfluids, vortex reconnections were first studied theoretically by Schwarz [1, 2], who pioneered the vortex filament method to study the dynamics of vortices in a superfluid. Later Levine et al. [3] studied the vortex reconnections using non-linear Schrödinger equation as a model for the superfluid system. Vortex reconnections have been experimentally observed in liquid Helium [4], whereas in Bose-Einstein condensates (BECs) their experimental realisation is yet to be accomplished. We study the vortex reconnections in phase-separated binary condensate of ^{85}Rb - ^{87}Rb [5]. In this system, the tunability of scattering length of ^{85}Rb [6] allows us to choose the appropriate interaction strength suitable for the creation of coreless vortices [7, 8], i.e phase-singularities in one of the species with the corresponding density maxima in the other [9, 10]. Using the phase imprinting method [] we examine the reconnections between six counter propagating coreless vortex rings, and between a coreless vortex ring and a coreless vortex line. For this we consider the system in a spherically symmetric trapping potential. It must be mentioned that all the previous studies on the vortex reconnections have considered only single species Bose-Einstein condensates (BECs). In earlier works, vortex reconnections has been studied between counter propagating rings, resulting in the cascade of Kelvin waves [11]. A similar study [12] reported the vortex reconnections between two rings, and a ring and a line. Further more, the vortex reconnections between two counter-propagating vortex rings with a slight offset between their axes and resulting in the emission of sound waves was studied in Ref. [13]. Coming to vortex lines, reconnections between the bundles of vortex lines in superfluid has been studied in Ref. [14]. All of these studies, however, are with homogeneous and single species superfluids. To the best of our knowledge, the present studies are the first studies on the vortex reconnections (between coreless vortices) in binary systems.

VORTEX RINGS IN CONDENSATES

In the mean field approximation, a scalar BEC is described by the Gross-Pitaevskii (GP) equation

$$\left[\frac{-\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + U|\Psi(\mathbf{r}, t)|^2 - i\hbar \frac{\partial}{\partial t} \right] \Psi(\mathbf{r}, t) = 0, \quad (1)$$

where $U = 4\pi\hbar^2 a/m$ with a as the s -wave scattering length is inter-atomic interaction parameter, and $V(\mathbf{r})$ is the trapping potential. In the present work, we consider ^{87}Rb condensate [15] with 10^6 atoms in a spherical trap

$$V(\mathbf{r}) = \frac{m\omega^2 r^2}{2}, \quad (2)$$

where $\omega/2\pi = 100\text{Hz}$ is the trapping frequency. The Eq. (1) can be written in scaled units using $a_{\text{osc}} = \sqrt{\hbar/m\omega}$, ω^{-1} , and $\hbar\omega$ as the units of length, time, and energy respectively. The scaled GP equation is

$$\left[-\frac{\tilde{\nabla}^2}{2} + V(\tilde{\mathbf{r}}) + \tilde{U}|\phi(\tilde{\mathbf{r}}, \tilde{t})|^2 - i\frac{\partial}{\partial \tilde{t}} \right] \phi(\tilde{\mathbf{r}}, \tilde{t}) = 0, \quad (3)$$

where $\tilde{\nabla}^2 = a_{\text{osc}}^2 \nabla^2$, $\tilde{U} = 4\pi a N/a_{\text{osc}}$, and $\phi(\tilde{\mathbf{r}}, \tilde{t}) = \sqrt{a_{\text{osc}}^3/N} \Psi(\mathbf{r}, t)$ with N as the number of atoms. For notational simplicity, we will represent the scaled quantities without tilde in the rest of the manuscript. The scaled wavefunction ϕ is normalised to unity.

We first study the reconnections between six vortex rings. The stationary state solution is obtained by propagating the coupled Eq. 3 in imaginary time. After each time step phase consistent with the presence of six vortex rings in ^{87}Rb is imprinted. Mathematically it implies that if r_0 is the radius of the six rings located at $x = \pm R_0$, $y = \pm R_0$, and $z = \pm R_0$, redefine $\phi(\mathbf{r})$ as

$$\phi = |\phi| \prod_{i=1}^3 \exp \left(i \tan^{-1} \frac{x_i - R_0}{\sqrt{\sum_{j=1}^3 x_j^2 - r_0}} \right) \prod_{i=1}^3 \exp \left(-i \tan^{-1} \frac{x_i + R_0}{\sqrt{\sum_{j=1}^3 x_j^2 - r_0}} \right), \quad (4)$$

after each time step. Here $x_1 = x$, $x_2 = y$, and $x_3 = z$ respectively and $j \neq i$. The stationary state solution thus obtained is shown in Fig. 1(a).

Vortex ring reconnections

The radius of the core of the vortex ring is approximately equal to the coherence length ξ . In the point source approximation, we assume the vortex ring to be infinitely thin, i.e. $\xi \rightarrow 0$. For such a ring with the center located at $(0, 0, 0)$ and lying on the xy

plane in an infinite homogeneous medium, the three components of the velocity field are

$$\begin{aligned} v_x &= \frac{\Gamma}{4\pi R} \int_0^{2\pi} \frac{z \cos \theta}{[1 + r^2 - 2(x \cos \theta + y \sin \theta)]^{3/2}} d\theta, \\ v_y &= \frac{\Gamma}{4\pi R} \int_0^{2\pi} \frac{z \sin \theta}{[1 + r^2 - 2(x \cos \theta + y \sin \theta)]^{3/2}} d\theta, \\ v_z &= \frac{\Gamma}{4\pi R} \int_0^{2\pi} \frac{1 - x \cos \theta - y \sin \theta}{[1 + r^2 - 2(x \cos \theta + y \sin \theta)]^{3/2}} d\theta, \end{aligned} \quad (5)$$

where Γ is the circulation, R is the radius of the ring, and $r = \sqrt{x^2 + y^2 + z^2}$ is the distance of the observation point from the center of the ring. Hence, the total velocity field is

$$\mathbf{v}(\mathbf{r}) = v_x(\mathbf{r})\hat{i} + v_y(\mathbf{r})\hat{j} + v_z(\mathbf{r})\hat{k}. \quad (6)$$

If there is another ring located at (t_x, t_y, t_z) on xy plane, the total velocity field produced by these two rings is

$$\mathbf{v}(\mathbf{r})_t = \mathbf{v}(\mathbf{r}) + \mathbf{v}(\mathbf{r} - \mathbf{t}). \quad (7)$$

In case the ring is located in an arbitrary plane with Euler angles α, β , and γ , the velocity field is

$$\mathbf{v}'(\mathbf{r}') = R^{-1}(\alpha, \beta, \gamma)\mathbf{v}(\mathbf{r}'), \quad (8)$$

here rotational matrix $R(\alpha, \beta, \gamma) = R_z(\alpha)R_y(\beta)R_z(\gamma)$ and $\mathbf{r}' = R(\alpha, \beta, \gamma)\mathbf{r}$. The $R_z(\phi)$ and $R_y(\phi)$ define the rotation of the coordinate system about z and y axis in three dimensional Euclidean space and are given as

$$R_z(\phi) = \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \text{ and } R_y(\phi) = \begin{pmatrix} \cos \phi & 0 & -\sin \phi \\ 0 & 1 & 0 \\ \sin \phi & 0 & \cos \phi \end{pmatrix}. \quad (9)$$

For a pair of rings with an arbitrary orientation with respect to each other, we can always choose the coordinate system in such a way that the origin lies at the center of one of the rings and the second ring with its center at (t_x, t_y, t_z) (say) lies on the plane with Euler angles α, β , and γ . The most general expression for the velocity field produced by such a pair of rings is

$$\mathbf{v}(\mathbf{r})_t = \mathbf{v}(\mathbf{r}) + R^{-1}\mathbf{v}(R(\mathbf{r} - \mathbf{t})), \quad (10)$$

where $R = R(\alpha, \beta, \gamma)$. In the case of BECs in traps, which are inhomogeneous finite sized system with non-zero size of the vortex cores, the velocity field of the condensate can be evaluated numerically using the quantum mechanical relation

$$v(\mathbf{r}, t) = -\frac{\phi^*(\mathbf{r}, t)\nabla\phi(\mathbf{r}, t) - \phi(\mathbf{r}, t)\nabla\phi^*(\mathbf{r}, t)}{2|\phi(\mathbf{r}, t)|^2}. \quad (11)$$

The velocity field can be extended to the case of six vortex rings approaching each other along the three axes. Due to the mutual force between the neighbouring vortices,

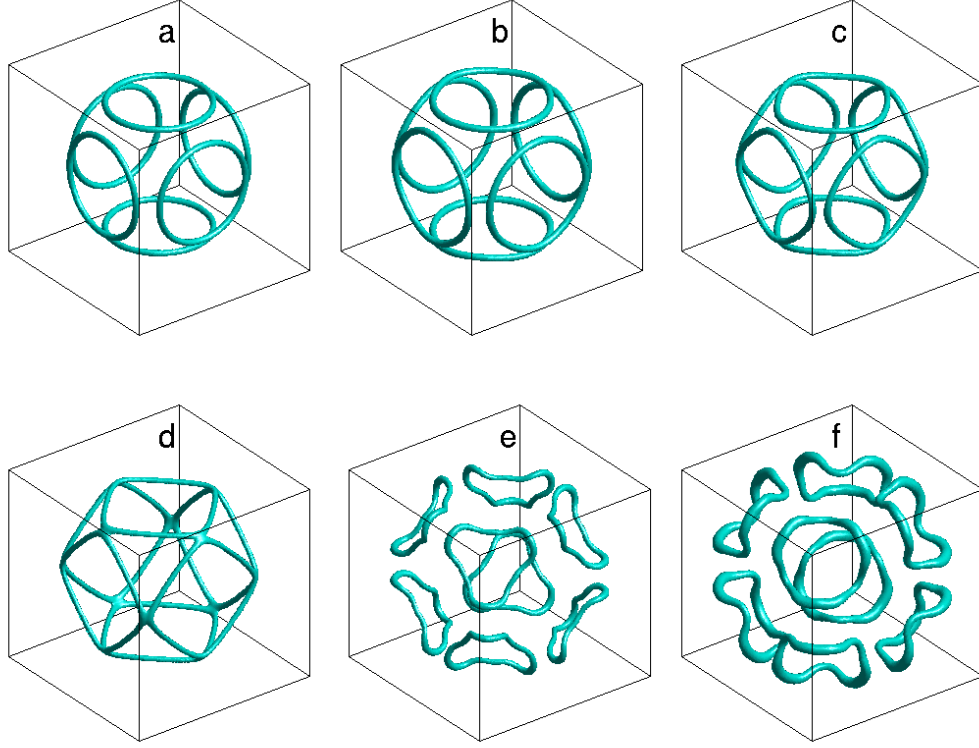


FIGURE 1. Reconnections between six vortex rings propagating in opposite directions along the three axes.

the portions of the rings along the orthogonal directions are distorted around the points of least separations. This stretches the vortex rings to a rectangular shape and this is clearly discernible in Fig. 1(b-c). The reconnection events sets in when the rings comes in contact (in Fig. 1(d)) and separates into eight smaller vortex rings after the reconnections, these are shown in Fig. 1(e-f). After the reconnections, the smaller rings propagate towards the periphery of the condensate and gets reflected back. To examine the nature of the reconnection events, we have also studied the vortex reconnections between four and two rings. The isosurfaces of ^{87}Rb with $|\phi_2(x,y,z)| = 0.015$ after the first reconnection event in these two cases are shown in Fig. 2. Topologically, there is a significant difference in the reconnection dynamics compared to the case of six vortex rings, however, there are several common qualitative features. One of the important pre-reconnection development is the stretching and distortion of the vortex rings. Regardless of the number of vortex rings, this is observed in all the reconnections,

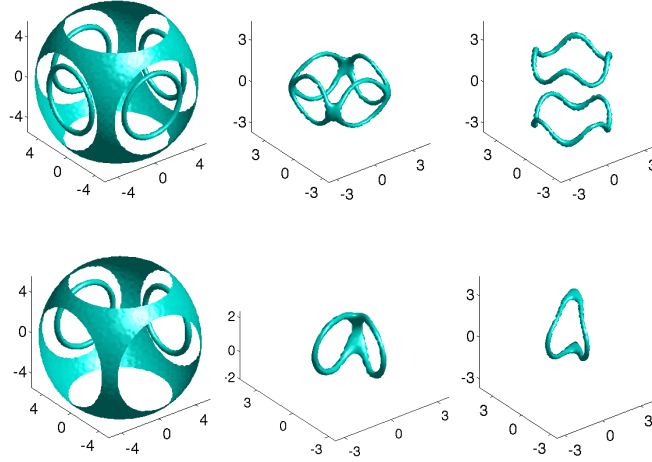


FIGURE 2. First reconnection event for four counter-propagating rings (upper row) and two orthogonally moving rings (lower row).

PHASE-SEPARATED BINARY CONDENSATE IN SPHERICAL TRAPS

As has been mentioned in the *introduction*, we consider phase-separated binary condensate of ^{85}Rb - ^{87}Rb in the spherically symmetric traps. The dynamics of this binary system at absolute zero can be studied by using coupled Gross-Pitaevskii (GP) equations

$$\left[\frac{-\hbar^2}{2m} \nabla^2 + V_i(\mathbf{r}) + \sum_{j=1}^2 U_{ij} |\Psi_j(\mathbf{r}, t)|^2 - i\hbar \frac{\partial}{\partial t} \right] \Psi_i(\mathbf{r}, t) = 0, \quad (12)$$

where $i = 1$ for ^{85}Rb and $i = 2$ for ^{87}Rb . Here $U_{ii} = 4\pi\hbar^2 a_{ii}/m_i$, where m_i is the mass and a_{ii} is the s -wave scattering length, is the intra-species interaction, $U_{ij} = 2\pi\hbar^2 a_{ij}/m_{ij}$, where $m_{ij} = m_i m_j / (m_i + m_j)$ is the reduced mass and a_{ij} is the inter-species s -wave scattering length, is the inter-species interaction, and $V_i(\mathbf{r})$ is the trapping potential for the i th species. Without loss of generality, we consider equal trapping potential for the two species, i.e.

$$V_i(\mathbf{r}) = \frac{m\omega^2}{2} (x^2 + y^2 + z^2), \quad (13)$$

where $m = m_1 \approx m_2$. In the present work, we consider $\omega = 2\pi 100.0\text{Hz}$ as the trapping frequency, $a_{11} = 450a_0$ [6], $a_{22} = 99a_0$ [16], and $a_{12} = 214a_0$ [17] as the scattering length values and $N_1 = 5 \times 10^5$ and $N_2 = 8 \times 10^5$ as the number of atoms. With these parameters, the ground state of the binary system is phase-separated with ^{85}Rb forming a shell around ^{87}Rb . The Eq. (12) can be written in scaled units using $a_{\text{osc}} = \sqrt{\hbar/m\omega}$, ω^{-1} , and $\hbar\omega$ as the units of length, time, and energy respectively. The scaled GP

equation is

$$\left[-\frac{\tilde{\nabla}^2}{2} + V(\tilde{\mathbf{r}}) + \sum_{j=1}^2 \tilde{U}_{ij} |\phi_j(\tilde{\mathbf{r}}, \tilde{t})|^2 - i \frac{\partial}{\partial \tilde{t}} \right] \phi_i(\tilde{\mathbf{r}}, \tilde{t}) = 0, \quad (14)$$

where $\tilde{\nabla}^2 = a_{\text{osc}}^2 \nabla^2$, $\tilde{U}_{ii} = 4\pi a_{ii} N_i / a_{\text{osc}}$, $\tilde{U}_{ij} = 4\pi a_{ij} N_j / a_{\text{osc}}$, and $\phi_i(\tilde{\mathbf{r}}, \tilde{t}) = \sqrt{a_{\text{osc}}^3 / N_i} \Psi_i(\mathbf{r}, t)$. For notational simplicity, we will represent the scaled quantities without tilde in the rest of the manuscript.

CORELESS VORTEX RING RECONNECTIONS

In binary condensates, the coreless vortices [7, 8] occur when the core of the vortices associated with one of the species is occupied by the other species. This is reminiscent of using Hydrogen as tracer of vortices in superfluid Helium [4] to observe vortex dynamics including reconnection events. To study the reconnections between coreless vortex rings, the stationary state solution is obtained by propagating the coupled Eqs. 14 in imaginary time. After each time step phase consistent with the presence of six vortex rings in ^{87}Rb is imprinted. Mathematically it implies that if r_0 is the radius of the six rings located at $x = \pm R_0$, $y = \pm R_0$, and $z = \pm R_0$, redefine $\phi_2(\mathbf{r})$ as

$$\phi_2 = |\phi_2| \prod_{i=1}^3 \exp \left(i \tan^{-1} \frac{x_i - R_0}{\sqrt{\sum_{j=1}^3 x_j^2 - r_0}} \right) \prod_{i=1}^3 \exp \left(-i \tan^{-1} \frac{x_i + R_0}{\sqrt{\sum_{j=1}^3 x_j^2 - r_0}} \right), \quad (15)$$

after each time step. Here $x_1 = x$, $x_2 = y$, and $x_3 = z$ respectively and $j \neq i$. The stationary state solution thus obtained is shown in Fig.3.

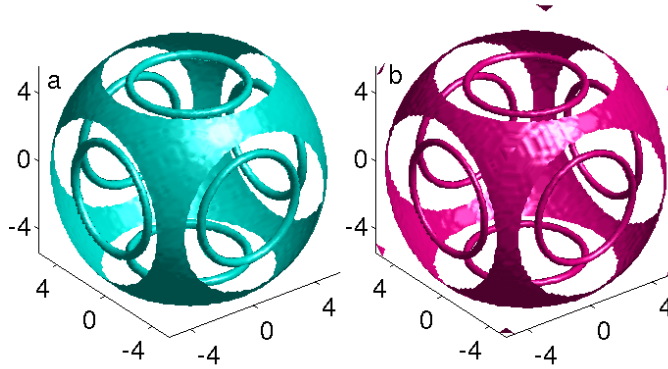


FIGURE 3. Isosurfaces of ^{87}Rb (left image) and ^{85}Rb (right image) with $|\phi_i(x, y, z)| = 0.015$. There are six coreless vortex rings in ^{87}Rb .

As the vortex rings approach each other, there are four distinct reconnection events. First, the six vortex rings start moving towards the center and undergo vortex reconnection at about 11 ms. Just prior to reconnections there is the stretching of vortex rings as is evidenced by the visual comparison of two images in the upper row of Fig. 4. This is consistent with previous theoretical studies. Like in the previous case of reconnections

in single species condensate, the reconnections leads to the formation of eight daughter vortex rings.

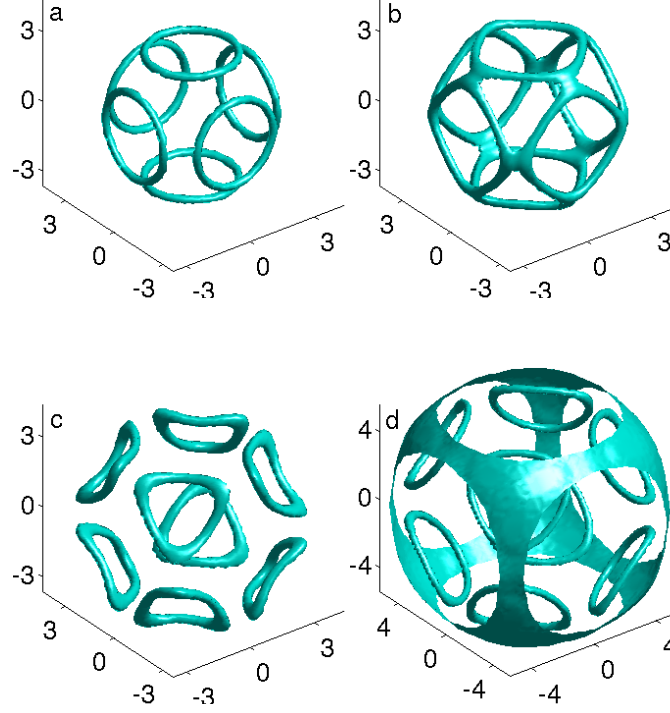


FIGURE 4. Isosurfaces of ^{87}Rb with $|\phi_2(x, y, z)| = 0.015$. There are six coreless vortex rings in ^{87}Rb which undergo vortex reconnections leading to formation of eight vortex rings. In upper row, the left image is at 8 ms and the right one is at 9 ms. The stretching of the vortex rings is evident from the images. In the lower row, the left image is at 10 ms, which shows the presence of Kelvin waves generated after reconnections, and the right one is at 15 ms.

In the second and third reconnection event, the eight daughter vortex rings formed after the first reconnection event move towards the interface, i.e. away from the the center of the trap. As they move away, there size increases. On the interface, the region where the two species overlap, there is a second reconnection event as is shown in the middle image in the upper row of Fig. 5. Again the resultant vortices undergo reconnections to form six small vortex rings; this is shown in the lower row of Fig. 5, where one complete ring and halves of the four other rings are shown in the rightmost image in the bottom row.

Finally, in the fourth reconnection event, the four small rings again start moving towards the origin and undergo reconnections after 23 ms. Like in the after first reconnection event, there is again the formation of eight smaller vortex rings which again start moving towards the interface. It is to be noted that the dynamics of the vortex reconnections in superfluids show one qualitative difference from the situation in viscous fluids. In the latter, the reconnection occurs through *bridging*, where the vortices after reconnection and separation retain a thin link [18] but this is absent in superfluids. An-

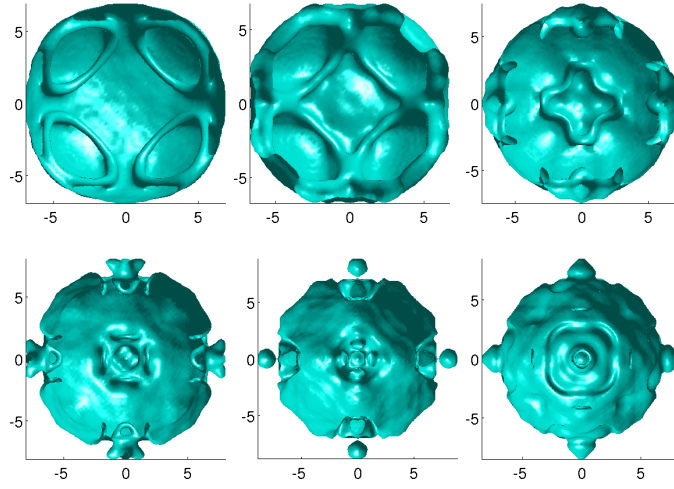


FIGURE 5. Half of the isosurfaces of ^{87}Rb with $|\phi_2(x, y, z)| = 0.015$. Middle images in upper and lower row show the second and third reconnections events taking place at 17 ms and 20 ms respectively.

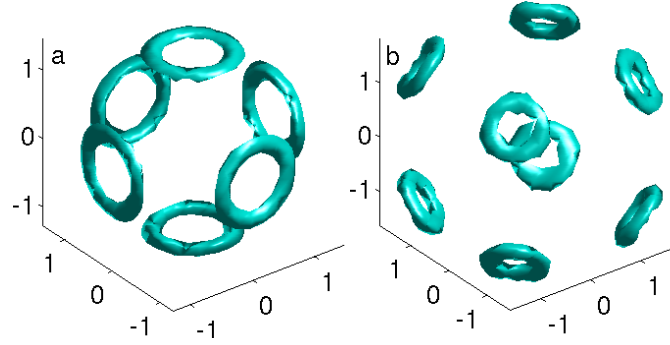


FIGURE 6. Isosurfaces of ^{87}Rb with $|\phi_2(x, y, z)| = 0.015$ at 23 ms (left image) and 24 ms (right image).

other point is, a dissipative process is prerequisite for vortex reconnections to occur in superfluids. So, in the BECs of trapped atomic gases we need to examine the origin and mechanism of the dissipative process. In BEC with inhomogeneous confining potentials, like in the present study, a vortex precess along equipotential surfaces. This leads to acceleration of the vortex elements and dissipates energy through acoustic radiation [19]. In the case of vortex rings, in addition to the self induced velocity of the ring, there is an acceleration arising from the inhomogeneity in the confining potential and this causes acoustic radiation. So, in superfluids the acoustic radiation is an important dynamical mechanism of energy dissipation and can play a fundamental role in vortex reconnections. In fact, one of our ongoing study is to examine this process in more detail. The other dynamical effect which could contribute to the dissipation is the interaction of the vortices with the background excitations. This, however, is likely to be less important for

the present work as the studies are at zero temperature. In conclusion, we have demonstrated that coreless vortex rings in binary condensates undergo vortex reconnections and in spherical geometry, there are multiple episodes of reconnections.

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